IMPROVED TREATMENT OF THE STRONGLY VARYING SLOPE IN FITTING SOLAR CELL I-V CURVES

A.R. Burgers J.A. Eikelboom A. Schönecker W.C. Sinke
Netherlands Energy Research Foundation ECN
PO Box 1 1755 ZG Petten The Netherlands
Tel: + 31-224-564703 Fax: + 31-224-563214 E-mail: burgers@ecn.nl

ABSTRACT

Straightforward least squares fitting of I-V curves leads to non optimal fits: residuals around and above the open-circuit voltage dominate the fit, leading to a bad fit at the maximum power point and lower voltage values. To deal with this problem authors have resorted to using weighting functions or to minimizing the area between data and fit instead of the least squares procedure. Both approaches lack a sound statistical basis.

Voltage noise has a big influence on fitting due to the steep slope of an I-V curve for higher voltage values. For this reason we have used Orthogonal Distance Regression (ODR), which is a mathematical method for fitting measurements with errors in both voltage- and current measurements. It allows for computing both the I-V curve parameters and their uncertainties.

INTRODUCTION

Measurement of the I-V curve of solar cells is one of the primary means of obtaining information about a solar cell. Useful parameters like the open circuit voltage V_{oc}, the short circuit current I_{sc} and the maximum power point voltage V_{mpp} and current I_{mpp} are easily obtained. When a good model for the I-V curve is available, more information can be extracted. I-V curves have been fitted in this work using the standard one- and two-diode models (1) and (2) respectively:

$$I(V) = \frac{V^*}{R_{sh}} + I_{01}\left[e^{\left(\frac{V^*}{n_1 V_b}\right)} - 1\right] + I_{lt} \quad (1)$$

$$I(V) = \frac{V^*}{R_{sh}} + I_{01}\left[e^{\left(\frac{V^*}{V_b}\right)} - 1\right] + I_{02}\left[e^{\left(\frac{V^*}{2V_b}\right)} - 1\right] + I_{lt} \quad (2)$$

Here I_{01} and I_{02} are the diode dark saturation currents, n_1 the ideality factor, I_{lt} the light generated current, V_b the thermal voltage and R_{sh} the shunt resistance. We will also use the shunt conductivity $G_{sh} = 1/R_{sh}$. The effect of series resistance is included:

$$V^* = V - R_{se} I(V) \quad (3)$$

Model parameters are determined in roughly two ways which will be dealt with in the following two subsections.

Curve characteristics

Easily determinable curve shape characteristics are determined by polynomial fits to small portions of the I-V curve. These curve shape characteristics are typically the short circuit current I_{sc}, the open circuit voltage V_{oc}, the curve slopes at the open circuit- and short circuit points, and the current I_{mpp} and voltage V_{mpp} at the maximum power point. A system of non-linear equations for the diode curve parameters parameters is set up which must be solved by numerical zero finding. Examples can be found in [1]- [3].

A disadvantage of this kind of methods is that not all measured points are used. It is also difficult to obtain error bounds on the parameters found. An advantage of these methods is the speed. As computers become faster, the aspect of speed becomes ever less important.

Fitting techniques

The second approach is to use fitting techniques. Let (I_i, V_i) be the measured current voltage values and $\vec{p}$ the vector of parameters to be determined. In case of the I-V curve models (1) and (2), $\vec{p}$ consists of G_{sh}, R_{se}, I_{lt}, I_{01} and n_1 or I_{02}. The optimum set of parameters $\vec{p}_o$ is determined by minimizing the following sum:

$$\vec{p}_o = \min_{\vec{p}} \sqrt[m]{\frac{1}{n}\sum_{i=1}^{n}\left(w_i\left|I_i - I(V_i, \vec{p})\right|\right)^m} \quad (4)$$

n is the number of data points measured. The w_i are a set of weights. m determines the norm used for fitting. For $m = 2$ we obtain standard least squares fitting. In least squares fitting the underlying assumption is that the current measurements are samples from a normal distribution with standard deviation $\sigma(I_i)$. The w_i should then be taken as $1/\sigma(I_i)$. In the case $w_i = 1$ the fit is unweighted and we implicitly assume all the current measurements are from the same distribution.

Straightforward unweighted least squares fitting of I-V data leads to non optimal fit parameters: residuals from the open-circuit onwards dominate the fit, leading to too large residuals near the maximum power point. A typical result is shown in Fig. 1.

To deal with this problem several authors have resorted to two basically different methods.

Usage of weights

In fitting dark current measurements it is common practice to use as weights the reciprocals of the measured dark current [4]- [6]. For illuminated I-V curves this choice of weights can not be used since the weights become too large near V_{oc} (Martinez [7]). Zdanowicz[8] proposes sev-

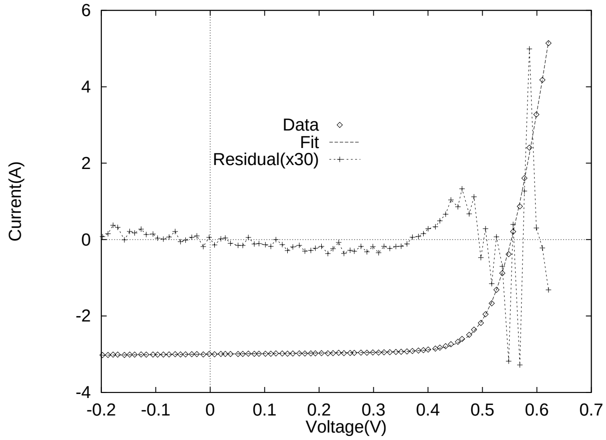

Fig. 1: Unweighted least squares fit to an I-V curve. The data points have been calculated with $I_{01} = 1. \times 10^{-9}A$, $I_{02} = 2. \times 10^{-5}A$, $G_{sh} = .1S$, $R_{se} = 7m\Omega$, $I_{lt} = -3A$. A voltage noise of 1mV and a current noise of 3mA was added.

eral weighting functions which give different fits. No clear arguments are presented however to chose between the different weight functions.

Usage of special norms

The higher the value of m is, the more sensitive the norm is to outliers. In the extreme case of $m \to \infty$, the m-norm corresponds to the maximum norm. In general the least squares or 2-norm is used because it can be motivated with statistical arguments if the errors in the measurements are assumed to be normal deviates.

Chan [9] introduced minimizing the surface area between data and fit which essentially is a fit in the 1-norm, with as weights the distances between the consecutive voltage values.

Cabestany [5] used besides the standard least squares fit the maximum norm $m \to \infty$.

Datta [10] used a mixed method for a single diode model. V_{oc} and I_{sc} are assumed to be known. This allows I_{lt} and I_{01} to be eliminated. The remaining parameters are determined by least squares.

Our work

Despite the large volume of literature on I-V curve fitting, we think an important aspect has been overlooked, being the influence of voltage noise as illustrated in Fig. 2.

The appreciable size of this effect is best illustrated with a numerical example based upon a 10x10cm^2 crystalline silicon solar cell under 1-sun illumination. We experience noise levels of around 3mA and 1mV in current and voltage respectively. The slope of the curve at V_{oc} typically is 20mΩ, so that the 1mV voltage noise causes a current noise level of 50mA, which is much larger than the 3mA error in the current measurement.

For this reason we have used Orthogonal Distance Regression (ODR). This is a mathematical method for fitting measurements with errors in both voltage- and current measurements. Both the choice of weights and the choice of other norms lack a sound statistical basis. ODR puts the fitted parameters on a sound statistical basis. We will describe this method and show results of the fits obtained.

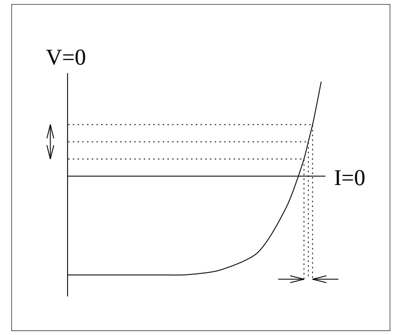

Fig. 2: A small voltage noise manifests itself as a large current noise due the steep slope of the I-V curve

ORTHOGONAL DISTANCE REGRESSION

There is a special method to find a fit in the case that there is both noise in current and voltage measurements. This method is known as Orthogonal Distance Regression (ODR) (see Boggs [11]) and can be used for instance for fitting a circle to a set of data points in a plane, indicating its ability to deal with strongly varying slopes.

We will designate noise levels in current and voltage with $\sigma(V)$ and $\sigma(I)$ respectively. With $\hat{V}_i$ and $\hat{I}_i$ we denote the fitted values.

The weighted ODR method finds a fit to the data by minimizing the following sum:

$$\min_{\hat{V}_i,\, \hat{I}_i,\, \vec{p}} \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{V_i - \hat{V}_i}{\sigma(V)}\right)^2 + \left(\frac{I_i - \hat{I}_i}{\sigma(I)}\right)^2\right)} \qquad (5)$$

under the n constraints that each pair $(\hat{V}_i, \hat{I}_i)$ has to be related by one of the diode-curve models from equations (1) and (2) in combination with (3). Note that for absence of voltage noise ($\hat{V}_i = V_i$), the ODR criterion (5) is equivalent to the standard least squares minimization. ODR assumes that noise in current- and voltage measurements are statistically mutually independent.

The name of the method originates from the fact that when voltage and current values are scaled with $\sigma(V)$ and $\sigma(I)$ respectively, the sum minimized by ODR consists of the sum of squares of the distances from the measured points to the noise scaled fitted I-V curve. The underlying completely statistical motivation of this method provides us with a simple criterion for the quality of the fit and allows for computing not only the diode curve parameters but also their uncertainties.

Charles [2] compared various methods of obtaining curve parameters through zero-finding on curve characteristics by looking at the sum of squares of distances from data points

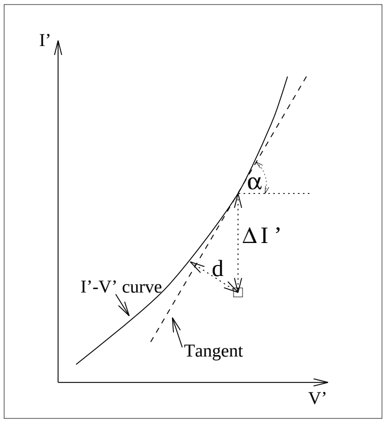

Fig. 3: An approximate value for the distance of a measured point (□) to the fitted noise scaled I-V curve (solid line) can be obtained from the current residual $\triangle I'$ at that point and the slope α of the curve

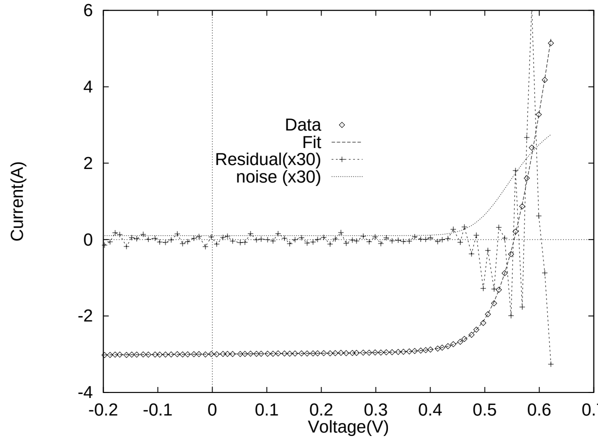

Fig. 4: Weighted least squares fit to the same data as in Fig. 1. The curve marked "Noise" refers to the total noise level from equation (6)

to fit. Both the short circuit current and the open circuit voltage were scaled to one. The ODR criterion was used in his work to compare fits obtained otherwise, not to fit with.

AN ODR BASED WEIGHTING SCHEME

In this section we will cast the ODR minimization procedure into a weighted least squares formulation. We observe that the total noise in a current observation consists from a part $\sigma(I)$ due to uncertainty in measuring the current and from a part $\frac{dI}{dV}\sigma(V)$ due to voltage uncertainty. We can compute a total noise level $\sigma_t(I(V))$ in the current measurement by adding the variances due to both noise sources:

$$\sigma_t(I(V))^2 = \sigma(I)^2 + \left(\frac{dI}{dV}\sigma(V)\right)^2 \tag{6}$$

The idea is that instead of a full ODR method we use a standard non-linear least squares routine to fit I as a function of V with $1/\sigma_t(I(V))$ as weights. In the following we show that the weighting scheme (6) can also be derived from ODR. The procedure is illustrated in Fig. 3.

We first transform current and voltage values as follows:

$$(I', V') = \left(\frac{I}{\sigma(I)}, \frac{V}{\sigma(V)}\right) \tag{7}$$

Let $\alpha(V')$ be the slope of the curve at voltage V'. $\alpha(V')$ can be related to the derivative of the $I(V)$ curve.

$$\tan(\alpha(V')) = \frac{dI'}{dV'} = \frac{\sigma(V)}{\sigma(I)}\frac{dI}{dV}$$

Let $\triangle I'$ be a residual in the transformed current. Because of the transformation we have $\triangle I = \sigma(I)\triangle I'$. We have for the distance d to the $I' - V'$ curve:

$$d \approx \cos(\alpha)\triangle I'$$

The approximation is exact in case of a linear model. Using the trigonometrical identity $\cos^2(x) = 1/(1 + \tan^2(x))$ we obtain for the squared distance d^2:

$$d^2 = \cos^2(\alpha)\triangle^2 I' = \frac{\triangle^2 I'}{1 + \tan^2(\alpha)} = \frac{\triangle^2 I}{\sigma(I)^2 + \sigma(V)^2 \frac{dI}{dV}^2} \tag{8}$$

Comparing with equation (6) we see that when the slope of the curve does not change to much, least squares fitting of I as a function of V with $1/\sigma_t(I(V))$ as a weight function is a good approximation to ODR. Also note that in practical solar cells, the slope of the curve is limited by series resistance, allowing this approach to be used. This approach can not be used in the same way to fit V versus I, because the shunt resistance can be large rendering the I-V curve flat for low voltage values.

The main advantage of the standard least squares formulation is that specialized Fortran routines (See Gay [12]) can be used which exploit the presence of linear parameters in the diode curve model, leading to faster and more reliable convergence. Initial estimates are needed only for the non-linear parameters. An initial estimate for the series resistance is easily obtained from the steepest slope that occurs, a good initial estimate for the ideality factor is 1.

Our computer code employs an iterative scheme such that the *fitted* current and not the measured current is used to calculate the effect of series resistance through equation (3).

RESULTS

Fig. 4 shows our ODR based fit to the same data as in Fig 1. Indeed the ODR fit follows the measurements more closely around the maximum power- and the short circuit point. Despite the large residuals for voltage above V_{oc}, the fit is still close to the data points owing to the steep slope of the curve. Fig. 4 also shows the total noise level $\sigma_t(I(V))$ computed with formula (6). Up to around the maximum power point at 475mV, our weighting scheme is the same as uniform weighting, since the curve is flat and errors in

Name	Uniform	
I_{lt}(A)	-3.0007	$(1 \pm 0.2\%)$
I_{01}(A)	0.12587×10^{-8}	$(1 \pm 3.7\%)$
I_{02}(A)	0.10572×10^{-4}	$(1 \pm 18.7\%)$
$R_{se}(\Omega)$	0.77580×10^{-2}	$(1 \pm 1.5\%)$
G_{sh}(S)	0.14337	$(1 \pm 16.7\%)$
	ODR	
I_{lt}(A)	-3.0001	$(1 \pm 0.0\%)$
I_{01}(A)	0.10410×10^{-8}	$(1 \pm 2.4\%)$
I_{02}(A)	0.18997×10^{-4}	$(1 \pm 3.5\%)$
$R_{se}(\Omega)$	0.71995×10^{-2}	$(1 \pm 2.0\%)$
G_{sh}(S)	0.10354	$(1 \pm 2.8\%)$

Table 1: Fitted curve parameters from Figs. 1 and 4 with their uncertainties

the voltage contribute little to errors in the current. From the maximum power point onwards, the influence of voltage noise increases sharply. This is explained by the fact that for higher voltages the total noise according to (6) can be approximated with:

$$\sigma_t(I(V)) = \frac{\mathrm{d}I}{\mathrm{d}V}\sigma(V) \tag{9}$$

Table 1 compares the results from both fits. The results can be compared with the known parameters of the I-V curve from Fig. 1. The ODR fit returns much better estimates of the I-V curve parameters and gives smaller and more accurate uncertainty levels. In the unweighted fit G_{sh} and I_{02} are most off.

A further conclusion, which is supported by calculations not shown here is that the fit is less sensitive to the range of voltage values used.

CONCLUSIONS

The strongly varying slope makes it difficult to fit solar cell I-V curves with standard least squares techniques. Orthogonal Distance Regression (ODR) is well suited to this fitting problem and puts the fitting procedure on a sound statistical basis. In a controlled way ODR allows residuals to be larger for voltage values above V_{oc}, thus obtaining a better fit for lower voltage values, while still maintaining a close fit above V_{oc}.

The uncertainties obtained in the parameters are more realistic. The χ^2 of the fit can be used to judge whether the one- or two-diode models are suitable. We have shown how ODR can be cast in the form of a weighted least squares formulation. The numerical procedure is fast and reliable enough to be used on-line.

ACKNOWLEDGMENTS

This work was carried out within the framework of the ECN ENGINE programme and the Dutch NWO priority programme "Solar Cells for the 21-st century"

REFERENCES

[1] J. C. H. Phang, D. S. H. Chan, and J. R. Philips, “Accurate analytical method for the extraction of solar cell model parameters”, *Electronics Letters*, vol. 20, no. 10, pp. 406–8, 1984.

[2] J-P Charles, I. Mekkaoui-Alaoui, G. Bordure, and P. Mialhe, “A critical study of the effectiveness of the single- and double exponential models for I-V characterization of solar cells”, *Solid State Electronics*, vol. 28, pp. 807–20, 1988.

[3] Quanxijia, W. A. Anderson, E. Liu, and S. Zhang, “A novel method for evaluating the series resistance of solar cells”, *Solar Cells*, vol. 25, pp. 311–8, 1988.

[4] M. Wolf, G. T. Noel, and R. J. Stirn, “Investigation of the double exponential in the current-voltage characteristics of silicon solar cells”, *IEEE Trans. on Electron Devices*, vol. ED 24, pp. 419–428, 1977.

[5] J. Cabestany and L. Castaner, “Evaluation of solar cell parameters by nonlinear algorithms”, *J. Phys. D*, vol. 16, pp. 2547–58, 1983.

[6] G. L. Araujo, E. Sanchez, and M. Marti, “Determination of the two-exponential solar cell equation parameters from empirical data”, *Solar Cells*, vol. 5, pp. 199–204, 1982.

[7] V. Martinez, R. Lizundia, and J. C. Jimeno, “Multiv: a computer program to fit I-V characteristics”, in *11th EC PSEC*, 1992, pp. 314–7.

[8] T. Zdanowicz, “The interactive computer program to fit I-V curves of solar cells”, in *Proceedings 12th European Photovoltaic Solar Energy Conference*. EEC, 1994, pp. 1311–4.

[9] D. S. H. Chan, J. R. Philips, and J. C. H. Phang, “A comparitive study of extraction methods for solar cell model parameters”, *Solid State Electronics*, vol. 29, pp. 329–37, 1986.

[10] S. K. Datta, K. Mukhopadhyay, S. Bandopadhyay, and H. Saha, “An improved technique for the determination of solar cell parameters”, *Solid State Electronics*, vol. 35, pp. 1667–73, 1992.

[11] P. T. Boggs, R. H. Byrd, and R. B. Schnabel, “A stable and efficient algorithm for non-linear orthogonal distance regression”, *SIAM J. Sci. Stat. Comput.*, vol. 8, no. 6, pp. 1052–78, 1987.

[12] D. M. Gay and L. Kaufman, “Tradeoffs in algorithms for separable nonlinear least squares”, in *IMACS ’91, Proceedings of the 13th World Congress on Computational and Applied Mathematics*, R. Vichnevetsky and J. J. H. Miller, Eds., Dublin, Ireland, 1991, pp. 157–158, Criterion Press.